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Agathe Keller

Expounding the Mathematical Seed

Volume 1: The Translation

A Translation of Bhāskara I on the Mathematical Chapter of the Āryabhaṭīya

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A Translation of Bhāskara I on the Mathematical Chapter of the Āryabhatīya

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Abreviations and Symbols

When referring to parts of the treatise, the *Āryabhaṭīya*, we will use the abbreviation: "Ab". A first number will indicate the chapter referred to, and a second the verse number; the letters "abcd" refer to each quarter of the verse. For example, "Ab. 2. 6. cd" means the two last quarters of verse 6 in the second chapter of the *Āryabhaṭīya*.

With the same numbering system, BAB refers to Bhāskara's commentary. Mbh and Lbh, refer respectively to the *Māhabhāskarīya* and the *Laghubhāskarīya*, two treatises written by the commentator, Bhāskara.

[] refers to the editor's additions;

⟨⟩ indicates the translator's additions;

() provides elements given for the sake of clarity. This includes the transliteration of Sanskrit words.

Introduction

This book presents an English translation of a VIIth century Sanskrit commentary written by an astronomer called Bhāskara. He is often referred to as Bhāskara I or "the elder Bhāskara" to distinguish him from a XIIth century astronomer of the Indian subcontinent bearing the same name, Bhāskara II or "the younger Bhāskara".

In this commentary, Bhāskara I glosses a Vth century versified astronomical treatise, the *Āryabhaṭīya* of Āryabhaṭa. The *Āryabhaṭīya* has four chapters, the second concentrates on *gaṇita* or mathematics. This book is a translation of Bhāskara I's commentary on the mathematical chapter of the *Āryabhaṭīya*. It is based on the edition of the text made by K. S. Shukla for the Indian National Science Academy (INSA) in 1976[1].

How to read this book?

This work is in two volumes. Volume I contains an Introduction and the literal translation. Because Bhāskara's text alone is difficult to understand, I have added for each verse's commentary a supplement which discusses the linguistic and mathematical matter exposed by the commentator. These supplements are gathered in volume II, which also contains glossaries and the bibliography. The two volumes should be read simultaneously.

This Introduction aims at providing a general background for the translation. I would like to help the reader with some of the technical difficulties of the commentary, in appearance barren and rebutting. My ambition goes also beyond this point: I think reading Bhāskara can become a stimulating and pleasant experience altogether.

The Introduction is divided into three sections. The first places Bhāskara's text within its historic context, the second looks at its mathematical contents, the third analyzes the relations between the commentary and the treatise.

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[1] The Bibliography is at the end of volume II, on p.227. The edition is listed under [Shukla 1976]. The conventions used for the translation are listed in the next section on p.1.

Let us start by describing Bhāskara's commentary: we will shortly observe where it stands within the history of mathematics and astronomy in India and specify, afterwards, the type of mathematical text Bhāskara has written.

A Situating Bhāskara's commentary

1 A brief historical account

The following is a short sketch of the position of Bhāskara's *Āryabhaṭīyabhāṣya* ("commentary of the *Āryabhaṭīya*", the title of his book) among the known texts of the history of mathematics and astronomy in India[2].

The oldest mathematical and astronomical corpus that has been handed down to us in this geographical area are related to the vedas.

The vedas are a set of religious poems. They are the oldest known texts of Indian culture. These poems also form the basis on which, later, Hinduism developed. This is probably why their date and origin are still subject to intense historical and philological debates[3]. These poems have been commented upon in all sorts of ways, grammatically, philosophically, religiously, ritually. The sum of these commentaries is called the *vedāṅga*s. There is a mathematical component to these texts which is related to the construction of the altars used in religious sacrifices. These mathematical writings are called the *śulbasūtra*s[4]. They are of composite nature, have different authors and thus several dates. The earliest is generally considered to be the *Baudhyānaśulbasūtra* of circa 600 B.C.[5] The *śulbasūtras* typically describe constructions with layered bricks or the delimitation of areas with ropes. They are not, however, devoid of testimonies of general mathematical reflections. For instance, they state the "Pythagoras Theorem".[6] Reading Bhāskara's commentary, one comes across objects and features (such as strings) that are inherited from this tradition[7].

Bhāskara's text, however, belongs to a different mathematical tradition. Indeed, the *śulbasūtra*s, together with the *vedāṅgajyotiṣa* (circa. 200 B. C.), an astronomical treatise with no mathematical content, are historically followed by a gap:

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[2]For more detailed accounts one may refer to [Pingree 1981], [Datta and Singh 1980], [Bag 1976]. Many books have been published in India on this subject, they usually recollect what was printed in the afore mentioned classics.

[3]The nature and scope of these debates have been analyzed in [Bryant 2001]. While most Indologists will agree to ascribe to the vedas the date of ca. 1500 B. C., traditional pandits and scholars with a bend towards hindu nationalism might quote very old dates, starting with 4000 or 5000 B. C. and going further back.

[4]All the edited and translated texts are gathered in [Bag & Sen 1983].

[5][CESS, volume 4].

[6]For more details see [Sarasvati 1979], [Bag 1976], [Datta & Singh 1980] and [Hayashi 1994, p. 118].

[7]The question of the posterity of the *śulbasūtras* in Bhāskara's commentary remains an area open for further investigation.

after that, the Hindu tradition[8] has not handed down to us any mathematical or astronomical text dated before the Vth century A.D.

At that time, two synthetic treatises come to light: the *Pañcasiddhānta* of Varāhamihira[9] and the *Āryabhaṭīya* of Āryabhaṭa[10]. The importance of the *Āryabhaṭīya* for the subsequent astronomical reflection in the Indian subcontinent can be measured by the number of commentaries it gave rise to and the controversies it sparked. Indeed no less than 18 commentaries have been recorded on the *Āryabhaṭīya*, some written as late as the end of the XIXth century[11]. Both texts are typical Sanskrit treatises: they are written in short concise verses. They are compendiums. Varāhamihira's composition, for instance, as indicated by its title which means "the five *siddhāntas*", summarizes five treatises[12].

Bhāskara, probably a marathi astronomer[13], has written the oldest commentary of the *Āryabhaṭīya* that has been handed down to us. Consequently, it is the oldest known Sanskrit prose text in astronomy and mathematics. According to his own testimony it was composed in 629 A.D.

Very little is known about Bhāskara and his life, only that he is also the author of two astronomical treatises in the line of Āryabhaṭa's school, the *Māhabhāskarīya* and the *Laghubhāskarīya*[14].

Other VIIth century mathematical and astronomical texts in Sanskrit have been handed down to us. Brahmagupta's treatise the *Brahmasphuṭasiddhānta*[15] would have been written in 628 A.D. and, in the latest critical asssessment of its datations, the *Bhakhshālī Manuscript*[16], a fragmentary prose, is also roughly ascribed to the VIIth century[17].

Thus, the VIIth century appears as the first blossoming of a renewed mathematical and astronomical tradition. Thereafter, a continuous flow of treatises and commen-

[8] The Hindu tradition is the sum of mathematical works developed by Hindu authors. It includes almost all the texts written in Sanskrit, although Hindu authors have also written in other dialects. Buddhists, for which we do not have any early testimony of mathematical writings, and Jain authors almost systematically wrote in their own dialects. At the beginning of the VIth century a council in Valabhī, a town mentioned in Bhāskara's examples, fixed the Jain canon which includes astronomical and mathematical texts. Although not written in Sanskrit, some quotations of these works are found in our commentary. These Jain texts testify to the existence of mathematical and astronomical knowledge developed outside of the Hindu tradition, prior to the VIIth century, and probably before the Vth century as well.

[9] [Neugebauer & Pingree 1971].

[10] [Sharma & Shukla 1976].

[11] [Sharma & Shukla 1976; xxxv-lviii].

[12] Concerning the name *siddhānta* for astronomical treatises, see [Pingree 1981].

[13] [Shukla 1976; xxv-xxx], [CESS; volume 4, p. 297].

[14] These texts have been edited and translated by K. S. Shukla: [Shukla 1960], [Shukla 1963]. They had also been previously edited with commentaries, see [Apaṭe 1946] and [Sastri 1957]. For more details one can refer to the entry Bhāskara in the [CESS, volume 4, p. 297-299; volume 5].

[15] [Dvivedi 1902].

[16] [Hayashi 1995].

[17] For a discussion of the time when the text would have been written, see [Hayashi 1995, p. 148-149].

taries in Sanskrit were produced and preserved. At that time, Sanskrit astronomical texts and knowledge spread outside the frontiers of the Indian subcontinent: by the IXth century, there where Indian astronomers at the Tang courts and most probably the first Indian treatises were translated into Arabic. The precise story of these astronomical and mathematical creations still needs to be written. However, they provide testimony to the florescence of these disciplines in India during this period. By the XVIIth century, in turn, texts in Arabic and Persian started to imprint their mark on the astronomical knowledge of India, announcing a new way of practicing this discipline.

Bhāskara's prose writing is therefore important because it provides information on the beginning of one of the richest moments in the development of mathematics and astronomy in ancient India. It can, indeed, furnish clues to the relations these mathematics have to the former tradition of Vedic geometry. Furthermore, Bhāskara's *Āryabhaṭīyabhāṣya* proposes an interpretation of an important Vth century treatise. We will see later that it is certainly *his reading* of this text. Furthermore, Bhāskara's commentary does not only shed a light on the treatise, it also provides detailed insights on the authors' own mathematical and astronomical practices.

2 Text, Edition and Manuscript

Bhāskara's mathematics are not unknown to historians of mathematics. An edition of his commentary was published in 1976 by K. S. Shukla for the Indian National Science Academy[18], the completion of a series that had started at the University of Lucknow in the 1960's with the publication of editions and translations of Bhāskara's two other astronomical treatises[19]. These were followed by a number of articles by the same author on Bhāskara's mathematics[20]. Books published in India will often refer to him for his contributions to the pulverizer and his arithmetics, if not for his trigonometry or his use of irrational numbers. Bhāskara is indeed famous and glorious, but nothing much is usually said beyond broad generalities. Among the reasons that could be ascribed to such an attitude, one should insist on the difficulties presented by the edited text itself. It is difficult to read.

2.1 On the edition and its manuscripts

This difficulty can be ascribed to the scarcity and state of the sources that were used while elaborating the edition.

[18] [Shukla 1976].
[19] [Shukla 1960] and [Shukla 1962].
[20] [Shukla 1971 a], [Shukla 1971 b], [Shukla 1972 a], and [Shukla 1972 b].

Indeed, only six manuscripts[21] of the commentary are known to us. Five of them were used to elaborate Shukla's edition. These five belong to the Kerala University Oriental Manuscripts Library (KUOML) in Trivandrum and one belongs to the Indian Office in London[22]. All the manuscripts used in the edition prepared by K. S. Shukla have the same source. This means they all have the same basic pattern of mistakes, each version having its own additional ones as well. They are all incomplete. Shukla's edition of the text has used a later commentary on the text inspired by Bhāskara's commentary, to provide a gloss of the end of the last chapter of the treatise. The fact that this edition relies on a single faulty source is probably one of the reasons why Bhāskara's commentary at times seems obscure or nonsensical. As many old Indian manuscripts still belong to private families or remain hidden in ill-classified libraries, one can still hope to find supplementary recensions that would enable a revision the edition.

While the lack of primary material is a major difficulty, other problems arise from the quality of the edition itself. K. S. Shukla has indeed performed the tedious meticulous work required for an edition. However some aspects of this endeavor, retrospectively, raise some questions. Let us first note that no dating of the manuscripts or attempt to trace their history has been taken up. Secondly, nonsensical or problematic parts have not been systematically pointed out and discussed. K. S. Shukla has indeed provided in many cases alternative readings. However, these are never justified and sometimes go contrary to the sensible manuscript readings that he gives in footnotes[23]. But in many cases, nonsensical sentences are found in the text without any comment at all. A third problem arises as editorial choices concerning textual arrangements (such as diagrams and number dispositions) are often, if not systematically, implicit. I have consulted four of the six manuscripts of the text and can testify that dispositions of numbers and diagrams vary from manuscript to manuscript. Discrepancies between the printed text and the manuscript further deepen the already existing gap between the written text and the manuscripts themselves. Concerning the latter, manuscripts and edition are separated by more than 1000 years of mathematical practices[24]. Consequently, all study of diagrams, or of *bindu*s as representing zero should be carried out carefully.

[21] Five of which are made of dried and treated palm leaves which were carved and then inked, a traditional technique in the Indian subcontinent. Palm-leaf manuscripts do not keep well, and thus Sanskrit texts have generally been preserved in a greater number on paper manuscripts.

[22] Shukla has used four from the KUOML and the one from the BO. A fifth manuscript was uncovered by D. Pingree at the KUOML. As one of the manuscripts of the KUOML is presently lost it is difficult to know if the "new one" is the misplaced old one or not. Furthermore, this manuscript is so dark that its contents cannot be retrieved anymore.

[23] As specified in the next section, p. 1, when this was the case, the translation adopted was that of the manuscript readings.

[24] A case study on the dispositions of the Rule of Three has been studied in [Sarma 2002] which underlines such discrepancies. Palmleaf manuscripts can not be much older than 500 years.

2.2 Treatise versus commentary

Bhāskara's fame is also obliterated by Āryabhaṭa's celebrity. Āryabhaṭa is a figure that all primary educated Indians know. He is celebrated as India's first astronomer. India's first satellite was named after him. This reputation rests upon the understanding we have of his works and achievements. As we will attempt to show later on, for this we need to rely on his commentators. And indeed, historically, many who achieved understanding of Āryabhaṭa have been indebted to Bhāskara. The first publication of Āryabhaṭa's text in Sanskrit[25] was accompanied by a commentary by Parameśvara, another astronomer and commentator on Āryabhaṭa. Parameśvara knew Bhāskara's commentary and relied on it. Subsequent translations in English and German have first relied on Parameśvara's commentary and, when it came to be known, on Bhāskara's commentary as well[26]. Bhāskara's importance can be measured by looking at the different understandings scholars (traditional and contemporary) have had of Āryabhaṭa's text. T. Hayashi has shown how Bhāskara's misreading of verse 12 of the mathematical chapter of the *Āryabhaṭīya*[27] has induced a long chain of misleading interpretations[28].

Why then, has the commentator been "swept under the rug", to use a French expression? The bias, privileging the treatise over its commentaries has partly its origin in the field of Indology itself. Indeed, even if we do not restrict ourselves to the astronomical and mathematical texts, the great bulk of Sanskrit scholarly literature is commentarial. Moreover, in India, commentaries could be as important as the treatises they glossed. For example, for the grammatical tradition, the *Māhabhaṣya* is probably as important as the text it comments, the *Aṣṭādhyāyi* of Pāṇini. However, despite their importance, there exists almost no thorough study on the genre of Sanskrit commentaries produced in a discipline whose object is after all ancient Indian texts[29]. A similar disregard of commentaries can also be found in the field of history of mathematics. Thus Reviel Netz's study of late medieval Euclidean commentaries, in an attempt to rehabilitate their importance, is not devoid of such prejudices[30]. The disregard of commentaries in both disciplines is probably a contemporary remanant of the Renaissance disregard for this kind of literature, a hint that these fields of scholarship were born in Europe. Whatever the reason, the consequence has been that the contents of Bhāskara's astronomical and mathematical texts has little been detailed in secondary literature.

Our aim is thus to focus on Bhāskara's work, highlighting two aspects: his interpretations of Āryabhaṭa's verses and his personal mathematical input. Let us

[25] [Kern 1874].

[26] See [Sengupta 1927], [Clark 1930], and [Sharma & Shukla 1976].

[27] From now on, all verses referred to belong to the mathematical chapter of the *Āryabhaṭīya*, unless otherwise stated.

[28] See [Hayashi 1997a].

[29] Let us nevertheless mention [Renou 1963], [Bronkhorst 1990], [Bronkhorst 1991], [Houben 1995] and [Filliozat 1988 b, Appendix], which are first attempts in specific disciplines, such as grammar, and at given times (Bronkhorst looks at the the VIIth century).

[30] See [Netz 1999] and as an answer [Chemla 2000], [Bernard 2003].

specify briefly what Āryabhaṭa's verses are and how the commentary is structured before giving an overview of its contents.

2.3 Āryabhaṭa's sūtras

The *Āryabhaṭīya* is composed of concise verses, mostly in the famously difficult *āryā* (the first chapter being an exception and being written in the *gītikā* verse[31]). These hermetic rules are known as *sūtra*s. Āryabhaṭa's *sūtra*s can be definitions (like verse 3[32] which defines squares and cubes) or procedures (like verse 4[33] which provides an algorithm to extract square roots). Some are a blend of such characterizations (thus verse 2[34] defines the decimal place value notation and the process to note such numbers). They manipulate technical mathematical objects such as numbers, geometrical figures and equations. Āryabhaṭa's *sūtra*s use puns, which gives to them an additional mnemonic flavor. Let us look, for instance, at Verse 4:

> One should divide, constantly, the non-square ⟨place⟩ by twice the square-root|
> When the square has been subtracted from the square ⟨place⟩, the quotient is the root in a different place||
>
> *bhāgaṃ hared avargān nityaṃ dviguṇena vargamūlena|*
> *vargād varge śuddhe labdhaṃ sthānāntare mūlam||*

As analyzed in the supplement on this verse and its commentary[35], the rule describes the core of an iterative process: the algorithm computes the square-root of a number noted with the decimal place value notation. It is concise in the sense that one needs to supply words to understand with more clarity what is referred to. This is indicated in the translation by triangular brackets (⟨⟩[36]). Its brevity is connected to a pun: one does not know if the "squares" referred to in the verse are square numbers or square places (a place corresponding to a pair/square power of ten in the decimal place value notation). Obviously, this pun has also a mathematical signification, providing a link between square places and square numbers. Even when Āryabhaṭa's verses do not handle such elaborate techniques, they often only state the core of a process. We often do not know what is required and what is sought, or what are all the different steps one should follow to complete the algorithm. Indeed, such rules call for a commentary.

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[31] For more precision on the form of the treatise, one can refer to [Keller 2000; I] or see [Sharma & Shukla 1976].

[32] See BAB.2.3, volume I, p. 13-18.

[33] See BAB.2.4, volume I, p. 20.

[34] See BAB.2.2, volume I, p. 10.

[35] See volume II, p. 15.

[36] Conventions for such symbols are listed in volume I, p. ix.

2.4 Structure of the commentary

Bhāskara's commentary follows a systematical pattern. This structure can be found in other mathematical commentaries as well[37]. He glosses Āryabhaṭa's verses in due order.

The structure of each verse commentary is summarized in Table 1.

Table 1: Structure of a verse commentary

Introductory sentence
Quotation of the half, whole, one and a half or two verses to be commented
General commentary, e.g.
Word to word gloss, staged discussions, general explanations and verifications
"Solved examples" (*uddeśaka*)
Versified Problem
"Setting-down" (*nyāsa*)
"procedure" (*karaṇa*)

Each verse gloss starts by an introductory sentence which gives a summary of the subject treated in the verse. This introduction is followed by a quotation of the verse(s) to be commented. It is succeeded by what we have called a "general commentary". This portion of the text is a word to word gloss of the verse, where syntax ambiguities are lifted, words supplied and technical vocabulary justified and explained. This is also the part of the commentary which will present staged dialogs and discussions justifying Bhāskara's interpretation of Āryabhaṭa's rule. This "general commentary" is followed by a succession of solved examples. Each solved example once again is molded into a quite systematical structure. It is first announced as an *uddeśaka*. It is followed by a versified problem. The versified problem precedes a "setting-down" (*nyāsa*), where numbers are disposed, diagrams drawn as they will be used on a working surface from which the problem will be solved. This is followed by a resolution of the problem called *karaṇa* ("procedure").

Having thus described Bhāskara's commentary and located it historically, let us now turn to its contents.

In the following section we will present a structural overview of the mathematics of Bhāskara's commentary. A second section will attempt to draw the attention of the reader to the characteristics of the *Āryabhaṭīyabhāṣya* as a mathematical *commentary* of the Sanskrit tradition.

[37] See for instance [Jain 1995], [Patte].

B The mathematical matter

The mathematical chapter of the *Āryabhaṭīya* contains a great variety of procedures, as summarized in Table 2 on page xx.

Subjects treated range from computing the volume of an equilateral tetrahedron (verse 6) to the interest on a loaned capital (verse 25), from computations on series (verses 19-22) to an elaborate process to solve a Diophantine equation (verse 32-33). All of these procedures are given in succession, without any structural comment. It is the commentator, Bhāskara, who introduces several ways to classify them[38]. We will take up one such classification that seems to contain a relevant thread to synthesize Āryabhaṭa's and Bhāskara's treatment of *gaṇita* (mathematics/computations [39]): namely the distinction between *rāśigaṇita* ("mathematics of quantities") and *kṣetragaṇita* ("mathematics of fields"[40]). Naturally, Bhāskara's "arithmetics" or "geometry" does not always distribute procedures into the categories we would expect them to be allotted to. For instance, rules on series are considered as part of geometry. Furthermore, these classifications are not exclusive and a procedure can bear both an "arithmetical" and a "geometrical" interpretation[41]. Let us insist here that we are considering Bhāskara's practice of mathematics as we know very little of Āryabhaṭa's mathematics.

We will follow the opposition between the categories of *rāśigaṇita* and *kṣetragaṇita* to list a certain number of characteristics of mathematics as practiced by Bhāskara. While doing so, we will underline the ambiguities and uncertainties that these subdivisions raise. Our stress will be on the *practices* of mathematics that Bhāskara's commentary testifies of. Having examined separately procedures belonging to "arithmetic" and to "geometry" in Bhāskara's sense, we will analyze what are the relations entertained by these two disciplines. We will then turn, to articulating the broader link of mathematics with astronomy.

1 Bhāskara's arithmetics

Let us first look at the quantities used by Bhāskara before examining some aspects of his arithmetical practices. These activities and objects belong to the commentary. Unless stated, they are not mentioned in the treatise.

[38] I have analyzed these classifications and the definition of *gaṇita* in [Keller forthcoming].

[39] This word is used to refer to the subject or field "mathematics" but can also name any computation. I have discussed this polysemy in [Keller 2000; volume 1, II. 1] and in [Keller forthcoming]. This is also briefly alluded to below, on p.xxxviii and in the Glossary at the end of volume II (p.197.)

[40] *Kṣetra*, "field", is the Sanskrit name for geometrical figures.

[41] This will be discussed in more detail below on p. xxxiv.

Table 2: Contents of the Chapter on mathematics (*gaṇitapāda*)

Verse 1	Prayer
Verse 2	Definition of the decimal place value notation
Verse 3	Geometrical and arithmetical definition of the square and the cube
Verse 4	Square root extraction
Verse 5	Cube root extraction
Verse 6	Area of the triangle, volume of an equilateral tetrahedron
Verse 7	Area of the circle, volume of the sphere
Verse 8	Area of a trapezium, length of inner segments
Verse 9	Area of all plane figures and chord subtending the sixth part of a circle
Verse 10	Approximate ratio in a circle, of a given diameter to its circumference
Verses 11-12	Derivation of sine and sine differences tables
Verse 13	Tools to construct circle, quadrilaterals and triangles, verticality and horizontality
Verses 14-16	Gnomons
Verse 17	Pythagoras Theorem and inner segments in a circle
Verse 18	Intersection of two circles
Verses 19-22	Series
Verses 23-24	Finding two quantities knowing their sum and squares or product and difference
Verse 25	Commercial Problem
Verse 26	Rule of Three
Verse 27	Computations with fractions
Verse 28	Inverting procedures
Verse 29	Series/First degree equation with several unknowns
Verse 30	First degree equation with one unknown
Verse 31	Time of meeting
Verses 32-33	Pulverizer (Indeterminate analysis)

1.1 Naming and noting numbers

There is a difference between the way one names a number with words, and the way it is noted, on a working surface, to be used in computations.

1.1.a Naming numbers Sanskrit uses diverse ways of naming numbers, Bhāskara resorts to many. There exist technical terms for numbers, which bear Indo-European characteristics: thus the name for digits are *eka, dva, tri, catur, pañca, ṣaḍ, sapta, aṣṭa, nava*. Some numbers can alternatively be named by operations of which they are the result, thus *ekonavaviṃśati* (twenty minus one) for nineteen or *trisapta* (three ⟨times⟩ seven) for twenty-one. Numbers, especially digits, can also be named by a metaphor which is indicative of a number. Thus, the moon (*śaśin*) refers to one. A pair of twin gods, the Aśvins, can name the number 2, etc. As the last example shows, most of these metaphors rest upon images that spring from India's rich mythological tradition. These metaphors are used essentially when giving very big integer numbers: the commentator then enumerates in a compound (*dvandva*) the digits that constitute the number when it is noted with the decimal place value system, by following the order of increasing values of power of tens[42]. This was probably a way to ensure that no mistake was made when the number was noted. All of these devices can also be used to give the value of a fraction.The variety and complexity with which numbers are named require a mathematical effort: they need to be translated into a form that enables them to be easily manipulated on a working surface. This probably explains why a rule is actually given explaining how to note numbers (Ab.2.2). A glossary of the names of numbers can be found in volume II[43].

1.1.b Decimal place value notation To write down numbers, Bhāskara uses the decimal place value notation that Āryabhaṭa defines in verse 2 of the chapter on mathematics. The commentator is well aware of the advantages that this notation has on other types of notations[44]. No procedures for elementary operations are given in the text[45]. However, the rules Bhāskara gives to square and cube higher numbers and Āryabhaṭa's procedures to extract square roots rest upon such a notation of numbers and uses its properties. It is therefore highly probable that the same held true for elementary operations. Units (*rūpa*) accumulated produce digits (*aṅka*) and numbers (*saṅkhyā*). The word *aṅka* means "sign" or "mark" and could therefore refer to the symbols used to note the digits rather than to their

[42] The digits are therefore enumerated in an order that is opposite to the one with which they are noted. All of this is discussed and detailed in [Keller 2000; I.2.2.1]. For specifications on how the numbers have been translated see the next section, p. 2.

[43] See volume II, p. 221.

[44] This can be inferred from the rather obscure opening paragraph of the commentary of verse 2. It raises questions as to whether another system for noting numbers was prevalent in India. See BAB.2.2, volume I, p.10.

[45] Later texts describe such operations, using the decimal place value notation in the algorithms.