
Modeling for Structural Analysis

Behavior and Basics

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Preface

This book is aimed at a wide audience, and it has ambitious goals. If you are a student, the goal is to provide you with a foundation for the classes that you are taking in structural analysis and structural design. If you are a young engineer, the goal is to help you understand what you are doing when you use a computer program for structural analysis, and to help you become a better engineer. If you are an experienced structural engineer, the goal is to help you keep things in a proper perspective. If you are a university professor who teaches structural analysis, the goal is to persuade you to change the way that you teach the subject. In short, the goal of this book is to change how structural analysis is perceived and taught.

At the same time, the scope of this book is rather narrow. It covers the basics of modeling for structural analysis, but does not include many details. It covers the Direct Stiffness Method of analysis, using physical explanations rather than formal theory. It covers both material nonlinearity and geometric nonlinearity in considerable depth, with emphasis on physical understanding not on theory or mathematics. It also puts structural analysis in its proper place, as a tool for use in structural design, not as an end in itself. This book does not consider structural analysis theory, or how to program structural analysis for a computer. It considers linear, nonlinear, static and dynamic analysis, but does not explain the analysis theories in detail. Many of the details are topics for future volumes. Throughout the book the emphasis is on physical understanding, not on formal theory or mathematics.

There is a reason for this approach. I have often heard it said that young engineers use computer programs blindly, without understanding what they are doing. This is probably true, and it is unfortunate. However, my experience tells me that young engineers are not to blame.

The problem, I believe, is that engineering students are trained to see structural analysis as some magical thing that can tell us everything we need to know about the behavior of a structure, with a high degree of accuracy. This is an illusion. Structural analysis is at best highly approximate, and any predictions about structural behavior that are made by a computer program should be viewed with skepticism. Structural analysis is not some magical thing. It is merely a tool to help with structural design, and a highly imperfect one.

I have also heard it argued that the developers of computer programs are to blame (not CSI, but some competitors). I disagree. A computer program for structural analysis is a tool, and like any tool its primary goal is to enhance productivity. The program developer's task is to produce the best possible tool. The engineer's job is to use it with skill. It is the job of somebody else to provide young engineers with the education and training that they need to develop the skills. What are these skills, and who is the "somebody else"?

The following are my opinions on the required skills.

- (1) For the vast majority of engineers the skills do not include writing a computer program to do structural analysis. This may have been a useful skill in 1975, when structural analysis programs had limited capabilities and often had to be augmented. It is not true with today's computer software, which can do some amazing things. Computer program development is now a task for specialists. For the vast majority of engineers the challenge is to *use* computer programs, not *develop* them.

It is, however, a valuable skill to write a program to process analysis results in a specialized way, using languages such as Matlab, Mathcad or Visual Basic. These are general purpose tools that most engineers should be familiar with and use routinely.

- (2) The skills do not include analyzing a structure using classical "hand" calculation methods such as Moment Distribution. I have heard it argued that students should learn Moment Distribution in order to develop a "feel" for structural behavior. On this point I emphatically disagree. Moment Distribution was an excellent tool in its day, but it is outdated and of only historical interest. I have not used Moment Distribution in decades, and it does little to develop "feel".

Nevertheless, some hand calculation skills are definitely valuable. Free body diagrams and equilibrium equations are extremely useful for understanding the flow of forces and for checking that the forces from a computer analysis satisfy equilibrium. The moment-area method is extremely useful for checking that the deflections from a computer analysis are reasonable. These methods, and some other simple techniques, are essential skills. They also help to develop "feel".

- (3) The skills (for most engineers) do not include a detailed understanding of "matrix methods". To begin with, there is no such thing as a

"matrix method". There is matrix notation (which is extremely useful and should be used routinely by all engineers), and there are matrix formulations of structural analysis methods. Most analysis methods can be formulated with or without matrices. If a method is of only historical interest when formulated without matrices, adding matrix notation does not make it modern or useful.

There is, however, one analysis method that relies heavily on matrices. This is the Direct Stiffness Method. Almost all computer programs for structural analysis are based on this method. All engineers who do structural analysis should have a basic understanding of this method. They do not, however, need to understand the mathematical details, and they do not need to be able to program it for a computer. The Direct Stiffness Method is a very physical process, and most engineers need to understand it only in physical, not mathematical, terms. This means understanding nodes, elements, degrees-of-freedom, the physical meaning of a stiffness coefficient and a stiffness matrix, how the stiffness matrices for the elements in a structure can be assembled into a structure stiffness matrix, the need to solve thousands of simultaneous equations, and how things can go wrong if the analysis model is poorly conceived. The theoretical and computational details need to be mastered only by the relatively few engineers who work on computer program development.

- (4) Understanding how structural components behave is an essential skill. For a component that is elastic (or more correctly, that can be assumed to be elastic for analysis purposes), the key property is the stiffness, or stiffness matrix. For beam and column components this is usually in terms of bending stiffness (EI), axial stiffness (EA) and possibly shear stiffness (GA). Most textbooks on structural analysis imply that these values are well defined and easy to calculate. In a real structure that is often not the case. For example, how does one calculate EI for a reinforced concrete beam that has substantial cracking, and where the amount of cracking varies along the beam length? How does one calculate EI for a reinforced concrete column where the amount of cracking depends on the axial force? How does one know whether shear deformations are important or can be ignored? Textbooks rarely address such issues.

Also, this is just for elastic analysis. In many cases, especially for earthquake motions, a structural component can be loaded beyond yield and become inelastic. How does the component behave? What aspects of the behavior are important for analysis and design? What

properties are needed to capture these aspects in an analysis model? How can values for these properties be estimated? Since the properties are probably not known accurately, how does one account for the uncertainty? These are important issues that are rarely addressed in structural analysis courses or textbooks.

- (5) The ability to set up an analysis model that captures the important aspects of structural behavior is an essential skill. It is also every bit as challenging academically as "matrix methods". Indeed, in my opinion modeling for analysis is more challenging academically than analysis theory.

Related to this, the ability to check computer results for consistency is an essential skill (this is "feel" – do the results look right?). Much of this skill develops with experience, but it can be taught. It can not, however, be developed by learning Moment Distribution or Matrix Methods. It can be developed much more effectively by analyzing structures on a computer, examining the results critically, doing "what-ifs" by varying the structure properties, specifying unrealistically large stiffnesses to see what happens, and so on. For most engineers this is much more useful than analysis theory.

- (6) Knowing how computer results are used for making design decisions is an essential skill. In a typical structural analysis textbook the end result is a deflected shape and a bending moment diagram. In practice this is only the beginning. The important thing is how the analysis results are used to support decision making for design. Students should understand that structural analysis is not an end in itself, but merely a tool for use in design.

If I am correct, and these skills are the most important, why are they not being taught? The following are my opinions on the cause of the problem.

- (1) There are three phases in structural analysis, namely "modeling" at the beginning, "interpretation" at the end, and "computation" in the middle. For most engineers the most important phases are modeling and interpretation. The least important phase is computation (which includes analysis theory as well as number crunching).
- (2) The computation phase is always handled by a computer program. The program developers take care of the theory (and of many other things, such as data management and graphics) and the computer crunches the numbers. Most engineers can treat a computer program

as a "black box" that takes a model of the structure at one end and produces "results" at the other. An engineer must have confidence that the computations are done correctly, and must have an overall understanding of how the computations are performed, but he or she does not need to be concerned with the computational details. For most engineers the most important phases are modeling and interpretation. These phases generally require human skills and intelligence, and generally are not handled well by computer programs. (Automated modeling and interpretation is done to some extent, and it is a goal of program developers. Some engineers may look forward to the day when all three phases are automated, but be careful what you wish for.)

- (3) This brings me to what I believe is the problem. Engineering students, in Universities around the world, are being taught almost exclusively "computation", with little attention being paid to "modeling" or "interpretation". The skills that students are being taught are not useful, and the skills that are useful are not being taught. The "somebody else" who is responsible for teaching the needed skills is the University Professor, and he or she is often not doing a very good job. It is relatively easy to teach computation, which is mainly theory, and professors are usually good at theory. It is harder to teach modeling and interpretation, and this is something that professors often do not do so well.

It does not help that there are few, if any, textbooks that deal with modeling and interpretation. This book does not cover all of the above skills, but it does fill some of the gap. It covers the behavior of structural components, the direct stiffness method, and the basic principles of modeling and interpretation. It is planned as the first in a series, with future volumes that consider element modeling in depth, and explain in detail the assumptions and procedures for linear, nonlinear, static and dynamic analyses.

This is not a textbook in the usual sense, with worked examples and problems to be assigned. Rather, it provides background information on behavior and modeling. In order to teach the sorts of skills that I have referred to, a course in structural analysis would need to use exercises such as the following.

- (1) Set up free bodies of a variety of types. Use equilibrium equations and the virtual displacements principle to solve equilibrium problems. Use free bodies to check the results of computer analyses, for example

the forces on a beam-to-column connection. Emphasize to students that there is no excuse for errors in free bodies and equilibrium.

- (2) Sketch deflected shapes, to get a feeling for how structures deform and how deformed elements fit together to produce the deflected shape for a structure.
- (3) Solve simple deflection problems. Also check that the deflections calculated by computer analysis are reasonable. I like the moment-area method, because it is physical. I also like to use simple standard results, such as $PL^3/3EI$ for the deflection of a cantilever beam with a load at the tip. I do not like the virtual forces principle (the "dummy unit load" method), because it is too much of a mathematical process rather than a physical one.
- (4) Require students to use a computer program for structural analysis, starting on the first day of the first analysis course. Set up linear elastic models for structures of a variety of types, vary the stiffnesses of the elements, and run computer analyses. See the effects of the changes, and explain these effects. This is, I believe, the best way to develop a "feel" for structural behavior.
- (5) As students develop modeling and interpretation skills, add nonlinear analyses with material and geometric nonlinearity, always emphasizing the modeling assumptions and requiring explanations of the behavior. Also add dynamic analyses. A first undergraduate course in structural analysis could progress as far as simple inelastic analysis. A second course should include dynamic analysis.
- (6) Show how analysis results are used for design. Emphasize that structural analysis is at best very approximate, and that it is not an end in itself but merely a tool to support design. Coordinate course material in analysis and design - all too often they are taught as independent disciplines.

The following are some additional points on this book:

- (1) The words "in the author's opinion" could be added in many places. These words have been omitted to avoid excessive repetition. Much of the book is simply the author's opinion. Disagreement is welcomed.
- (2) There is no list of references. The task of compiling one and making the appropriate citations is simply too great. Since all analysis will be

done by computer, the best initial reference is the computer program documentation. This will lead to additional references.

- (3) This book considers design as well as analysis, and it references design codes and standards of practice. These are mainly the U.S. codes for steel and concrete, including ANSI/AISC 360 and ACI 318, and also ASCE 7 and ASCE 41. These are all well known and readily available. The notation in this book does not follow the notation in any particular code.
- (4) I would like to mention one book that is a rare example of a textbook that deals with structural behavior. This is *The Elements of Structure* by W. Morgan (edited by I. Buckle), second edition, 1977, Pitman. That book covers structural behavior, not modeling. However, successful modeling starts with an understanding of behavior. If you can find a copy of this book, it is well worth reading.

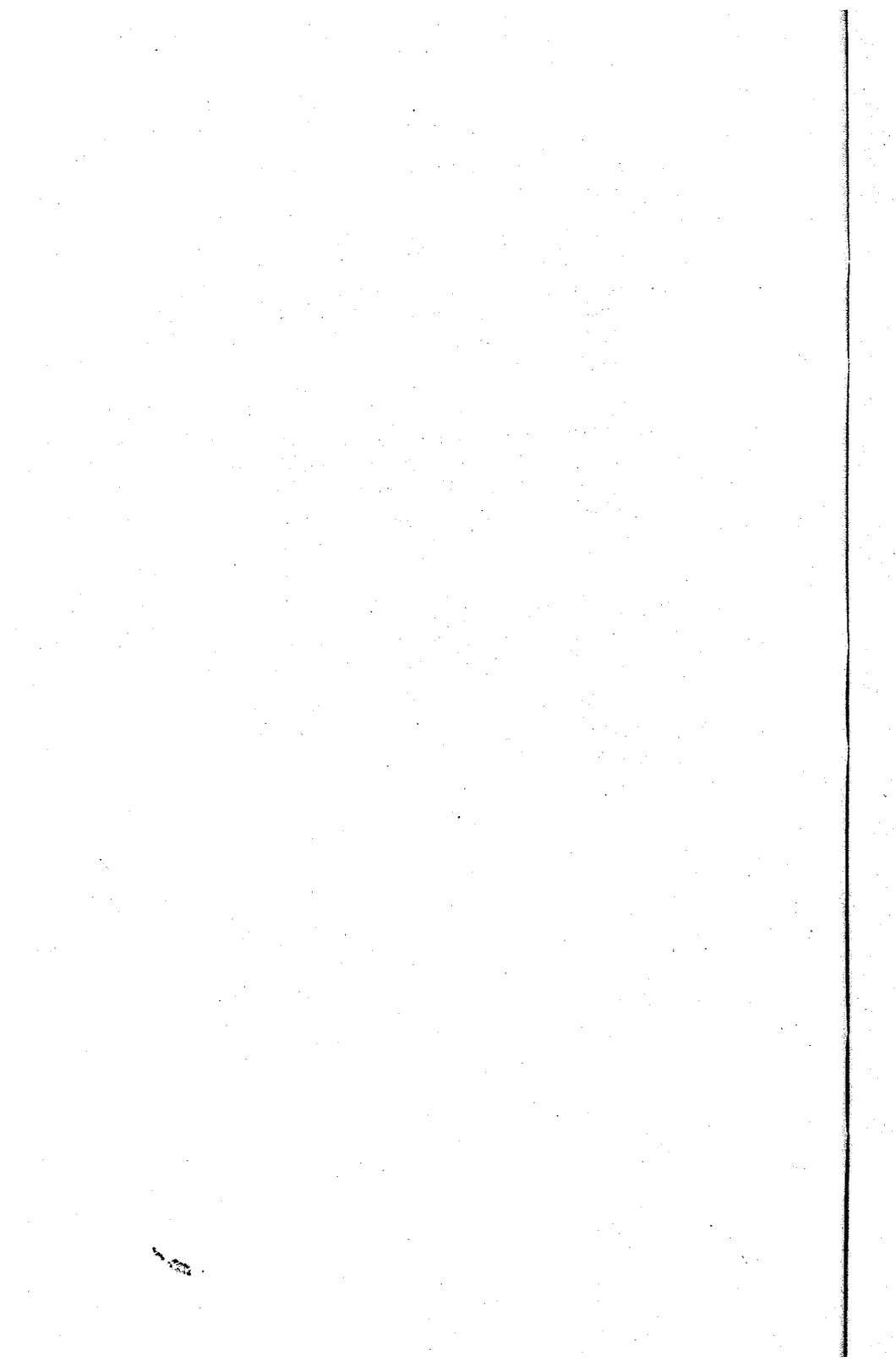
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Finally, I would like to dedicate this book to the memory of Professor Tom Paulay, the father of Capacity Design, the best teacher that ever was, and the nicest person I ever met.

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